QUESTION ONE

a) Explain the following terms as used in Markov analysis:
   i. Matrix of transition probabilities (2 marks)
   ii. Absorbing state (2 marks)
   iii. Recurrent state (2 marks)

b) Highlight four applications of functions in business. (4 marks)

c) ATC Limited manufactures an electric component whose marginal cost function is estimated to be quadratic in nature. The table below shows the marginal cost incurred by the company at three different production levels.

<table>
<thead>
<tr>
<th>Production (units)</th>
<th>Marginal Cost (Sh.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>22</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>200</td>
<td>436</td>
</tr>
</tbody>
</table>

The unit price associated with selling $q$ units of the component is given by:

$$P = 4.28 - 0.005q \text{ (in Sh. Million)}$$

Required:
   i) The marginal cost function (4 marks)
   ii) The total cost function, given that the fixed cost of production is Sh. 10 million (2 marks)
   iii) The number of units of the component that should be produced and sold in order to maximize profit. (4 marks)

QUESTION TWO

a) Describe the two laws of probability (4 marks)

b) Excellent Computers Limited deals in the production ribbons. Out of the total number of customers who use the company's printer ribbons in a given year, 20 percent use competitor's ribbons the following year. Out of the total number of customers who use the company's printer ribbons in a given year, 30 percent switch to Excellent Computers Limited's ribbons the following year. As at December 2014, Excellent Computer Limited had a total market share of 55 percent.

Required:
   i) The transition matrix (1 mark)
   ii) Excellent Computer Limited's total market share as at 31 December 2015 (2 marks)
   iii) Excellent Computer Limited's long run market share. (3 marks)

C) A bank has one counter to serve customers who have complaints about their loans. An average of 96 customers arrive at the counter in an 8-hour day. The customer service staff at the counter spend irregular amount of time servicing the arrivals which have been found to have an exponential distribution. The average service time per customer is 4 minutes.

Required:
   i) The average number of customers in the system. (2 marks)
   ii) The average number of customers waiting to be served (2 marks)
   iii) The average time a customer spends in the system. (1 mark)
   iv) The average queue waiting time. (2 marks)
   v) The probability that a customer will receive immediate service upon arrival (1 mark)
v) The probability that the number of customers in the system is greater than three. (2 marks)

QUESTION THREE

a) The following information relates to Alpha Limited's profit and associated dividend per share growth rates in percentage over the last ten years:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit growth rate %</td>
<td>7.6</td>
<td>4.4</td>
<td>-0.2</td>
<td>4.6</td>
<td>3.7</td>
<td>9.5</td>
<td>8.9</td>
<td>-0.3</td>
<td>0.9</td>
<td>6.1</td>
</tr>
<tr>
<td>Dividend growth rate %</td>
<td>6.8</td>
<td>4.1</td>
<td>3.1</td>
<td>4.2</td>
<td>8.2</td>
<td>7.9</td>
<td>5</td>
<td>3.9</td>
<td>6.1</td>
<td></td>
</tr>
</tbody>
</table>

Additional information:

\[ (y - \bar{y})^2 = 11.0657 \]

Where: \( y \) = observed values of \( y \)
\( \bar{y} \) = values calculated using the regression line.

Required:

i) The coefficient of determination. Interpret your result. (6 marks)

ii) The standard error of estimate. (2 marks)

iii) The estimated lower and upper dividend growth rates when the profit growth rate is 5.5 per cent. (6 marks)

b) Maxsoft Computer Systems Limited has been awarded a contract to design and install a management information system for Naivasha Enterprises limited. The table below is a summary of activities required to complete the project, normal time of each activity, shortest time an activity, shortest time an activity can be completed and the cost power day for reducing the time of each activity.

Required:

The total additional cost would be incurred if the project is completed within the shortest within the shortest time possible. (6 marks)

QUESTION FOUR

a) Outline four applications of set theory in business. (4 marks)

b) A wholesaler of a certain industrial product X orders the product four times in a year. Demand for product X is stable at 350,000 units per year. An order is placed when the stock of product X reaches 25,000 units, representing a 9-day working supply plus safety stock. The wholesaler has a 300-day working year. The wholesaler's cost data indicate that cost per order is Sh. 4,132 and annual unit carrying cost of product X is Sh. 20.76. The cost of a stock out is Sh. 5,400 due to the need to request for special orders from alternative suppliers. The probability of a stock out at various safety stock levels is as follows:

<table>
<thead>
<tr>
<th>Safety stock levels (units)</th>
<th>7,000</th>
<th>14,000</th>
<th>21,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of stock out</td>
<td>0.5</td>
<td>0.1</td>
<td>0.02</td>
</tr>
</tbody>
</table>

A probability of stock out per order of 0.02 is assumed in determining the order quantity. The company has a capacity to 90,000 units and order sizes are restricted to lots of 5,000 units.

Required:

i) The present total cost of the wholesaler's inventory policy. (3 marks)
 iii) The optimal order size (4 marks)

 iv) The re-order point under the optimal order size ans safety stock system. (2 marks)

c) Mary Mumbi has recently acquired a water pump which has a useful life of 10 years. It is estimated that the pump is likely to have periodic failures over its useful life. Past data of similar pumps indicate a probability distribution of failures as follows:

<table>
<thead>
<tr>
<th>Number of failures</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.75</td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Required:

Simulate the average annual failure rate of the pump using the random numbers, 65,80,30,15,50,90,52,73,62,92 (3 marks)

QUESTION FIVE

a) In relation to game theory, briefly explain the two rules of dominance. (2 marks)

b) The following data show the sales of sodas made by Tamu Soda Distributors Limited over a 10-week period:

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sale of Soda (cases)</td>
<td>220</td>
<td>230</td>
<td>250</td>
<td>240</td>
<td>250</td>
<td>240</td>
<td>260</td>
<td>230</td>
<td>220</td>
<td>260</td>
</tr>
</tbody>
</table>

Required:

i) Using exponential smoothing, advise Tamu Soda Distributors Limited on whether to use a smoothing constant of a=0.1 or a=0.8 in forecasting sales. (5 marks)

ii) Forecast sales for week 11 using a 3-week moving average. (1 mark)

iii) The following data relate to raw materials required for the production of each unit of two products, A and B.

<table>
<thead>
<tr>
<th>Raw materials requirements (kilogrammes)</th>
<th>Product A</th>
<th>Product B</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Y</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

The available quantities of raw materials X and Y are 300 kilogrammes and 400 kilogrammes respectively. The unit contributions of products A and B are Sh. 100 and Sh.40 respectively.

Required:

i) Using the simplex method, determine the number of units of products A and B that should be produced in order to maximize revenue. (11 marks)

ii) Determine whether your results in (c) (i) above would be different if full utilization of the available resources was assured. (1 mark)